Multi-mode Trajectory Optimization for Impact-aware Manipulation

Theodoros Stouraitis*[,1,2], Lei Yan*[,1], João Moura[1], Michael Gienger[2], and Sethu Vijayakumar[1]
[1] Institute of Perception, Action and Behaviour, University of Edinburgh, Edinburgh, EH8 9AB, U.K.
[2] Honda Research Institute Europe, Carl-Legien-Straße 30, 63073 Offenbach am Main, Germany.

***Abstract*—The transition from free motion to contact is a challenging problem in robotics, in part due to its hybrid nature. Yet, disregarding the effects of impacts at the motion planning level might result in intractable impulsive contact forces. In this paper, we introduce an impact-aware multi-mode trajectory optimization (TO) method that comprises both hybrid dynamics and hybrid control in a coherent fashion. A key concept is the incorporation of an explicit contact force transmission model in the TO method. This allows the simultaneous optimization of the contact forces, contact timings, continuous motion trajectories and compliance, while satisfying task constraints. We compare our method against standard compliance control and an impact-agnostic TO method in physical simulations. Further, we experimentally validate the proposed method with a robot manipulator on the task of halting a large-momentum object.**

I. INTRODUCTION

Safe and robust robot manipulation under switching dynamics still poses many challenges. Typically, manipulation tasks require making and breaking contact with objects. This results in challenges in motion planning and control due to, among other factors, (i) the hybrid nature of the problem [1] and (ii) the uncertainties that arises due to contact dynamics [2].

Recent hybrid Trajectory Optimization (TO) methods in robotics [3, 4, 5] have demonstrated efficient methods for multi-contact manipulation planning. Yet, it is not trivial to transfer these behaviours robustly on to the hardware due to the challenge of regulating the transitions between free motion and motion in contact, as well as dealing with imprecise timing of the transition in the reference motions. To address this, a number of hybrid control [6, 7] and compliance control [8, 9] methods have been proposed. However, given the inherent limitations of the hardware [10], the impacts that a stand-alone controller is capable of dealing with, are limited.

In this work, we try to address this problem at the level of ’impact-aware’ manipulation planning. We ask ourselves, *”How could we plan hybrid motions, such that they are easily tractable by out-of-the-box controllers?”*, which can be reframed as a problem of planning such that contact can be maintained during and after impact – even for tasks with contacts at speed, i.e. moving objects. As a typical example, consider an agent that stops an object in motion, as shown in Fig. 1. In such a case, the agent needs to address the following challenges:

- Plan discontinuous motions through contact. Contact events might trigger impacts, that result in state-triggered velocity jumps described by jump maps [11]. Thus, solving the problem on how to holistically select continuous motions (flows) and jumps for a task, is not trivial.
- Track discontinuous reference motions, where the actual time of the jumps (impact) cannot be assumed to coincide with the jump time (impact time) of the reference motion.

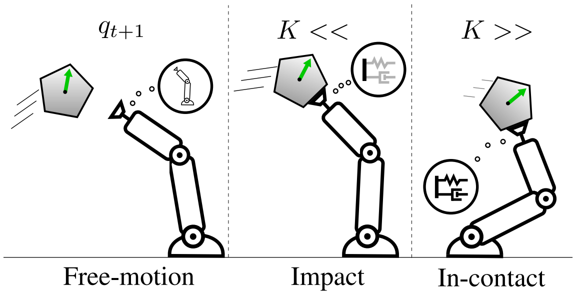

Fig. 1. Pictorial description for the task of halting a moving object.

A number of motion planning methods have investigated impact related problems. In [12, 13] impacts were avoided by specifying near-zero contact velocities. Catching was demonstrated based on learned dynamical systems [14] and with TO method [15], yet the intercepted objects have negligible mass.

Here, we address these two challenges with a coherent contact-invariant TO method that plans ’impact-aware’ hybrid motions, while the control input yields from a hybrid controller capable to absorb impacts. The hybrid controller is based on compliance control that allows to mitigate the peak error due to the mismatch in time between reference and actual impact. Our TO method results in hybrid motions that are inline with the hybrid controller, while the controller’s parameters (*e.g.* stiffness) are also optimized, as in [6]. By modulating the robot’s end-effector compliance, we can emulate a number of different types of collisions ranging from elastic to in-elastic, and deduce the optimal force transmission model given the system’s limitations, *e.g.* workspace limits.

II. IMPACT-AWARE MANIPULATION

Toussaint et al. [3] associate the notion of a mode with the “ contact activity”, i.e. change in contact state (physical interaction between objects). In this work, the behaviours of the investigated system, depend both on the contact state and the controller used. Thus, we refer to a single combination of a contact state and a controller as a mode of the system.

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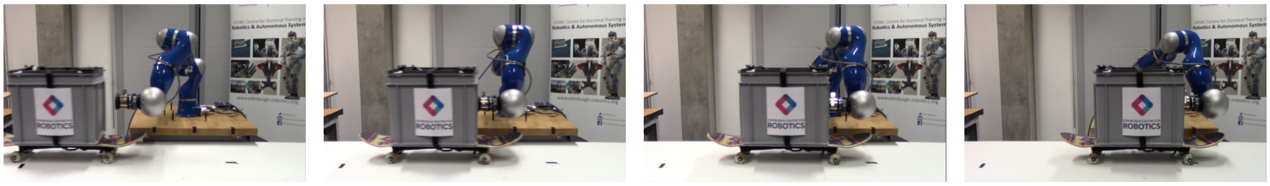
* Denotes equal contribution



(a) (b) (c) (d)

Fig. 2. Keyframes of the experiment where the robot halts a moving object with speed of 0.66m/s.

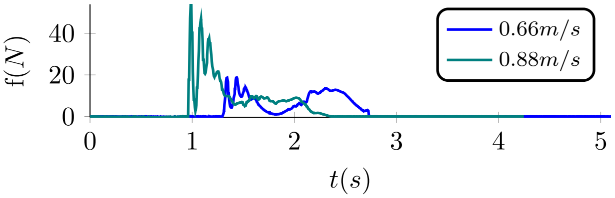


Fig. 3. Experimental result of contact force during halting motion.

A sequence of these contact-control modes $\boldsymbol{z} : \{z_0, z_1, ...z_J\}$ specifies the regime of optimal trajectories. Inspired by [3, 5, 16], we model impact-aware manipulation planning as a special form of Parametric Programming (PP) [17], as

$$\min_{\boldsymbol{x}(t),\boldsymbol{u}(t),\boldsymbol{v}(t)} \quad \boldsymbol{c}\left(\boldsymbol{x}(t)\boldsymbol{u}(t), \boldsymbol{v}(t), \boldsymbol{z}\right) \tag{1a}$$

$$\text{s.t.} \quad \dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{v}(t), \boldsymbol{z}\right), \tag{1b}$$

$$\dot{\boldsymbol{v}}(t) = \tilde{\boldsymbol{h}}\left(\boldsymbol{u}(t), \boldsymbol{z}\right), \tag{1c}$$

$$\boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{v}(t), \boldsymbol{u}(t), \boldsymbol{z}) \leq 0. \tag{1d}$$

where the sequence of modes $\boldsymbol{z}$ act as the selection variables in the peacewise functions (1a) - (1d), (1a) defines the objective function, (1b) is the system's hybrid dynamics, and $\boldsymbol{g}(\cdot)$ in (1d) represents both the equality and the inequality constraints of the system. This formulation includes both hybrid dynamics and hybrid control. We validate this formulation in the task of halting a large momentum object.

Based on [18, 19] the characteristics of a physical system—that transits from free-motion to contact—such as duration of impact and restitution coefficient, can be related to the mass, damping and stiffness parameters of the mass-spring-damper system. We utilize this observation to accurately emulate the physical interaction through the impedance controller of the manipulator. Further, for impact-aware manipulation we define two stages; (i) the deformation stage where the stiffness should be minimized to establish stable contact, and (ii) the restitution stage where the stiffness should be maximized to realize manipulation forces for tasks, such as pushing an object.

These stages are encoded in (1) in the form of controllers. In this way, the controller parameters (stiffness) and the trajectory are optimized, to conform with the different stages of contact. To realize this and to plan smooth contact forces, we model the force transmission as a second-order critically damped dynamical system (cd-DS), also used in [20]. We formulate a cd-DS for contact force transmission as

$$\ddot{\mathbf{f}}(t) + 2\alpha\dot{\mathbf{f}}(t) + \alpha^2\mathbf{f}(t) = \alpha^2\mathbf{f_d} \tag{2}$$

where the contact force $\mathbf{f}(t)$ satisfies $\mathbf{f}(t) \in [0, \mathbf{f_d}]$, while $\dot{\mathbf{f}}(t)$ and $\ddot{\mathbf{f}}(t)$ are its first and second derivatives. For any $\alpha > 0$, this contact force is critically damped. By enforcing the second-order contact force transmission model (2) to have the same characteristics as mass-spring-damper system, i.e. the same natural frequency ω_n and damping ratio ζ, we can obtain the following relationship between parameter α and the parameters of the mass-spring-damper system model.

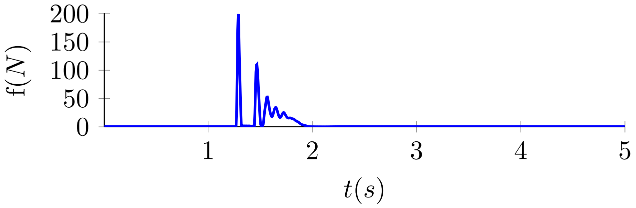


Fig. 4. Experimental result of impact between the object and the end-effector during halting an object with speed of 0.66 m/s.

$$\alpha = \sqrt{\frac{K}{M}}, \tag{3}$$

$$B = 2\sqrt{MK}. \tag{4}$$

Thus, given the mass of the object and the stiffness parameter we can obtain the cd-DS parameter α and vice versa.

III. ROBOT EXPERIMENTS

We validate our approach in a real setting with the KUKA LWR arm and the Vicon motion capture system, where the latter is used to measure the position of the object in real time. The experimental setup is shown in Fig. 2. The object is 20 kg and its velocity and acceleration are estimated on-the-fly. These values are then passed on to the impact-aware TO method, which computes an optimal motion plan in less that 150 ms—by solving an Non-Linear Program (NLP)—to halt the object within the workspace limits. The position and stiffness profiles of the motion plan are streamed to the robot, such that, the joint position with cartesian stiffness control mode of the KUKA LWR arm tracks the optimal motion.

In Fig. 3 we report contact force measured by an ATI F/T sensor. The object travels at the speed of 0.66 m/s. The proposed method halts its motion with the maximum force being less than 20 N. As a baseline we report the measured force with a very soft configuration (stiffness $K = 10$ and damping ratio $\lambda = 1$) of the LWR arm's standard compliance controller. In this case, the maximum impact force is 199.47 N (see Fig. 4), which is 10 times larger than the one shown in Fig. 3. Furthermore, to emphasize the capabilities of the method we consider the same object travelling at speed of 0.88 m/s with the respective contact force shown in Fig. 3.

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